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AN HYSTERESIS LOOP IN THE TWO COMPONENT BÉNARD PROBLEM

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Abstract—An approximation to the nonlinear theory of the two component Bénard problem, taking into account thermal diffusion, shows that multiple stable steady states are possible. The stability of each steady state is investigated. The nonlinear equations are then integrated numerically. An hysteresis loop in the Rayleigh number–Nusselt number plane is observed.

NOMENCLATURE

A, B, C, D, E, Fourier coefficients;
D, isothermal diffusion coefficient;
D', thermal diffusion coefficient;
d, depth of the liquid layer;
g, acceleration due to gravity;
N_i, mass fraction of component i;
N_i^*, initial mass fraction of component i;
Nu, Nusselt number;
n, perturbation of N;
Pr, Prandtl number;
Ra, Rayleigh number;
R_{Th}, Rayleigh number for the concentration field;
r, wavenumber in the horizontal direction;
Sc, Schmidt number;
$\mathscr{S}$, Soret number;
T, temperature;
t, time;
x_i, space coordinate.

Greek symbols

α, thermal expansion coefficient;
γ, $\rho^{-1}(\partial\rho/\partial N)_T$;
κ, thermal diffusivity;
ν, kinematic viscosity;
ρ, density;
ϑ, temperature perturbation;
ψ, stream function;
ϕ, vorticity.

1. INTRODUCTION

THIS paper considers the problem of a two fluid mixture confined between slippery horizontal walls which are heated from below. The system is initially homogeneous in composition, but, owing to the imposed temperature gradient, a mass fraction distribution is established in the liquid layer; this is the so-called Soret effect.

Several theoretical papers are devoted to the linear stability analysis [1–9]. The aim of the linear stability analysis, is to give the variation of the critical Rayleigh number as a function of a parameter, describing the influence of thermal diffusion. This parameter is called the Soret number. The main result is that for negative Soret numbers (i.e. when the denser component migrates towards the hot plate, here the lower boundary), the critical Rayleigh number increases: this is a stabilizing effect. A destabilizing effect is observed for positive Soret numbers when the denser component migrates towards the upper cold boundary. It was also found that under certain circumstances, the principle of exchange of stabilities was violated and that instability arises as oscillations of increasing amplitude (overstability).

Several papers were also devoted to an experimental investigation of the same problem [3,4,8,10–15]. In most of these papers, Schmidt–Milverton plots [16] are presented for an initially homogeneous two component system. Anamalous heating curves for negative Soret numbers are obtained showing a negative slope in a certain region (ΔT decreases when the heating power increases).

The situation is represented in Fig. 1. The main subject of this paper is to explain finite amplitude convection below the critical temperature gradient.

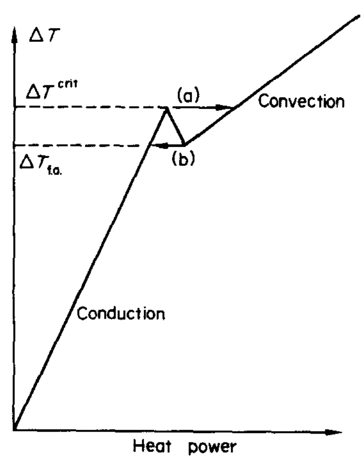

FIG. 1. Schmidt–Milverton plots for a two component system with negative thermal diffusion factor.

Referring to Fig. 1, if the Rayleigh number (or ΔT) is increased beyond its critical value, there is a sudden jump in the Nusselt number, indicated by arrow (a). If the Rayleigh number is now decreased from its maximum value, convection states extend below the critical ΔT. At a critical temperature gradient called $\Delta T_{\text{f.a.}}$, there is a jump back to the conduction regime ($Nu = 1$). This hysteresis loop can only be explained in the framework of a nonlinear theory. In order to obtain approximate solutions to the nonlinear equations with free boundary conditions, a truncated Fourier development is used. This method is described in an earlier work by Veronis [17, 18] and by Foster [19] on Bénard type problems.

2. THE NONLINEAR EQUATIONS

For an incompressible fluid, the conservation equations are

$$Sc\frac{\partial n}{\partial t} = Sc\frac{\partial(\psi,n)}{\partial(x,z)} + Sc.\mathscr{S}.\frac{\partial\psi}{\partial x} + \frac{\partial^2 n}{\partial x_j^2} + \mathscr{S}.\frac{\partial^2\vartheta}{\partial x_j^2} \tag{1}$$

$$\frac{\partial\phi}{\partial t} = \frac{\partial(\psi,\phi)}{\partial(x,z)} - \frac{Ra}{Pr}\frac{\partial\vartheta}{\partial x} + \frac{R_{Th}}{Pr}\frac{\partial n}{\partial x} + \frac{\partial^2\phi}{\partial x_j^2} \tag{2}$$

$$\phi = \frac{\partial^2\psi}{\partial x_j^2} \tag{3}$$

$$Pr\frac{\partial\vartheta}{\partial t} = Pr\frac{\partial(\psi,\vartheta)}{\partial(x,z)} - Pr\frac{\partial\psi}{\partial x} + \frac{\partial^2\vartheta}{\partial x_j^2} \tag{4}$$

with

$$\frac{\partial(f,g)}{\partial(x,y)} = \frac{\partial f}{\partial x}.\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}.\frac{\partial g}{\partial x}.$$

In these equations, ψ is the stream function, ϑ and n respectively the contribution of the convective state to the temperature field T and the mass fraction distribution N. Thus

$$\begin{aligned} T(x,z,t) &= \overline{T}(z) + \vartheta(x,z,t)\\ N(x,z,t) &= \overline{N}(z) + n(x,z,t)\\ \psi(x,z,t) &= 0 + \psi(x,z,t)\end{aligned} \tag{5}$$

with

$$\overline{T}(z) = 1 - z \quad \text{and} \quad \overline{N}(z) = 1 + \mathscr{S}(z - \tfrac{1}{2}).$$

All the variables are expressed in a dimensionless form. The notation is that adopted in the papers by Platten and Legros (e.g. 6)

Pr = Prandtl number $= \nu/\kappa$
Sc = Schmidt number $= \nu/D$
(D: isothermal diffusion coefficient)
Ra = Rayleigh number $= g\alpha\Delta T d^3/\kappa\nu$
R_{Th}: thermal diffusion Rayleigh number $= g\gamma N_1^* d^3/\kappa\nu$

$$\gamma = \frac{1}{\rho}\left(\frac{\partial\rho}{\partial N_1}\right)_T$$

$\mathscr{S}$: the Soret number $= (D'/D).\Delta T$
(D': thermal diffusion coefficient).

3. AN APPROXIMATE SOLUTION

Once convection has set in, the mean temperature and concentration field are distorted by the convective motions. Following Veronis [17, 18] the minimal representation which takes account of the finite amplitude motion is

$$\psi(x,z,t) = A(t)\sin\pi rx.\sin\pi z \tag{6}$$

$$\vartheta(x,z,t) = B(t)\cos\pi rx.\sin\pi z + C(t)\sin 2\pi z \tag{7}$$

$$n(x,z,t) = D(t)\cos\pi rx.\sin\pi z + E(t)\sin 2\pi z. \tag{8}$$

Let us recall that in the linear stability theory, infinitesimal perturbations are the form

$$\psi \sim \sin\pi rx.\sin\pi z; \quad \vartheta \sim \cos\pi rx.\sin\pi z \quad n \sim \cos\pi rx.\sin\pi z. \tag{9}$$

By substitution of (6)–(8) into the nonlinear equations (1)–(4) we deduce the following set of ordinary nonlinear coupled differential equations for the five Fourier time dependent coefficients

$$-\pi^2(r^2+1)\frac{dA}{dt} = \frac{Ra}{Pr}\pi rB - \frac{R_{Th}}{Pr}\pi rD + \pi^4(r^2+1)^2 A \tag{10}$$

$$Pr\frac{dB}{dt} = -Pr.\pi^2 rAC - Pr.\pi rA - \pi^2(r^2+1)B \tag{11}$$

$$Pr\frac{dC}{dt} = -4\pi^2 C + Pr.\pi^2 r\frac{A.B}{2} \tag{12}$$

$$Sc\frac{dD}{dt} = -Sc\pi^2 rA.E + Sc\mathscr{S}\pi rA - \pi^2(r^2+1)D - \mathscr{S}\pi^2(r^2+1)B \tag{13}$$

$$Sc\frac{dE}{dt} = -4\pi^2 E - 4\pi^2\mathscr{S}.C + Sc\pi^2 r\frac{AD}{2}. \tag{14}$$

The algebraic equations for the steady state ($dA/dt = dB/dt = \ldots = 0$) can be solved for one of the coefficients, say A. We get

$$\begin{aligned} A.\Bigg\{&\left(\frac{A^2}{8}\right)^2 Sc^2 Pr^2\pi^4(r^2+1)^2 r^4 + \left(\frac{A^2}{8}\right)\\ &\times\left[-r^4 Sc^2 Ra + \pi^4(r^2+1)^3 r^2(Pr^2+Sc^2)\right]\\ &+\bigg[-(r^2+1)r^2 Ra - R_{Th}\mathscr{S}(r^2+1)r^2\left(\frac{Sc+Pr}{Pr}\right)\\ &+\pi^4(r^2+1)^4\bigg]\Bigg\} = 0 \end{aligned} \tag{15}$$

and by back-substitution

$$B = \frac{-8rPr.A}{8\pi(r^2+1) + \pi r^2 Pr^2 A^2} \tag{16}$$

$$C = \frac{1}{8}Pr.r.A.B \tag{17}$$

$$D = [Ra.r.B + \pi^3(r^2+1)^2 Pr.A]/R_{Th}.r \tag{18}$$

$$E = \frac{Sc}{8}r.A.D - \mathscr{S}.C. \tag{19}$$

The Nusselt number, horizontally averaged, (< >) computed at the lower boundary ($z = 0$), is simply

$$<Nu>_{z=0} = 1 - 2\pi C$$

$$1 + \frac{2Pr^2(A^2/8)}{3 + Pr^2(A^2/8)}. \tag{20}$$

We need here some results of the linear theory [2] in order to rewrite equation (15).

$$r^2_{\text{crit}} = 1/2$$

$$Ra^{\text{crit}}_{\text{ex}} = Ra^{(0)} - R_{Th}.\mathscr{S}.\left(\frac{Sc+Pr}{Pr}\right) \tag{21}$$

where we have used the critical value for r and where $Ra^{\text{crit}}_{\text{ex}}$ is the critical Rayleigh number in the case of "exchange of stabilities" ($Ra^{(0)}$ being the usual Rayleigh number, i.e. $27\pi^4/4$); using equation (21), equation (15) can be rearranged:

$$A.\left\{\left(\frac{A^2}{8}\right)^2 Pr^2Sc^2.Ra^{(0)} - \left(\frac{A^2}{8}\right) \times 3Sc^2\left[Ra - Ra^{(0)}\left(1+\frac{Pr^2}{Sc^2}\right)\right] - 9(Ra - Ra^{\text{crit}}_{\text{ex}})\right\} = 0. \tag{22}$$

The solutions are

$$A = 0 \tag{23}$$

$$\frac{A^2}{8} = \frac{3}{2}\frac{1}{Pr^2}\left[\frac{Ra}{Ra^{(0)}} - \left(1+\frac{Pr^2}{Sc^2}\right)\right] \pm \sqrt{\left\{\frac{9}{4}\frac{1}{Pr^4}\left[\frac{Ra}{Ra^{(0)}} - \left(1+\frac{Pr^2}{Sc^2}\right)\right]^2 + \left(\frac{Ra - Ra^{\text{crit}}_{\text{ex}}}{Ra^{(0)}}\right).\frac{9}{Sc^2Pr^2}\right\}}. \tag{24}$$

The question is to know if real positive values of A^2 may exist below the critical point, thus if finite amplitude instability exists (real A).

Two cases must be considered:

(i) If $\mathscr{S} > 0$, then equation (21) shows that

$$Ra^{\text{crit}}_{\text{ex}} < Ra^{(0)} \quad \text{(a destabilizing case)}$$

and consequently, for

$$Ra < Ra^{\text{crit}}_{\text{ex}} < Ra^{(0)}$$

no positive values for A^2 can be obtained from equation (24), and the only solution is $A = 0$ corresponding to the state of rest ($Nu = 1$).

(ii) If $\mathscr{S} < 0$, it was shown previously [2] that overstability prevails if

$$|\mathscr{S}| > \mathscr{S}^* = \frac{27\pi^4}{4}\frac{Pr(Pr+1)}{Sc^2R_{Th}}. \tag{25}$$

For values of the dimensionless numbers compatible with experiments in liquids (e.g. water–isopropanol [8])

$$\mathscr{S}^* \simeq 10^{-6}$$

and experiments operate always beyond this value.

In that case, let us recall that

$$Ra^{\text{crit}}_{\text{over}} < Ra^{\text{crit}}_{\text{ex}}$$

where $Ra^{\text{crit}}_{\text{over}}$ is the critical Rayleigh number related to overstability.

Thus the main question is not to know if steady motions may exist below $Ra^{\text{crit}}_{\text{ex}}$, itself greater than $Ra^{(0)}$, thus for

$$Ra^{(0)} < Ra < Ra^{\text{crit}}_{\text{ex}}$$

but rather, if finite amplitude instability arises in the range

$$Ra^{(0)} < Ra < Ra^{\text{crit}}_{\text{over}}$$

i.e. below the first critical point encountered when the temperature gradient is increased.

The solution for $(A^2/8)$ was tabulated from equation (24) in a wide range of Rayleigh numbers; the other relevant parameters were kept constant, and equal to

$$Pr = 10$$
$$Sc = 1000$$
$$R_{Th} = 40\,000$$
$$\mathscr{S} = -10^{-2}$$

and thus

$$Ra^{\text{crit}}_{\text{over}} = 1028{\cdot}35 \quad \text{(see [2])}.$$

From equation (20) the Nusselt number can be computed. Figure 2 reproduces the Nusselt number vs the Rayleigh number. Curve (a) corresponds to the solution $A = 0$ of equation (22), i.e. the state of rest, whereas curve (b) corresponds to nonzero real values of A. We have indeed found three real solutions of equation (22) for A if

$$Ra \geqslant 760{\cdot}4$$

thus clearly below the critical point.

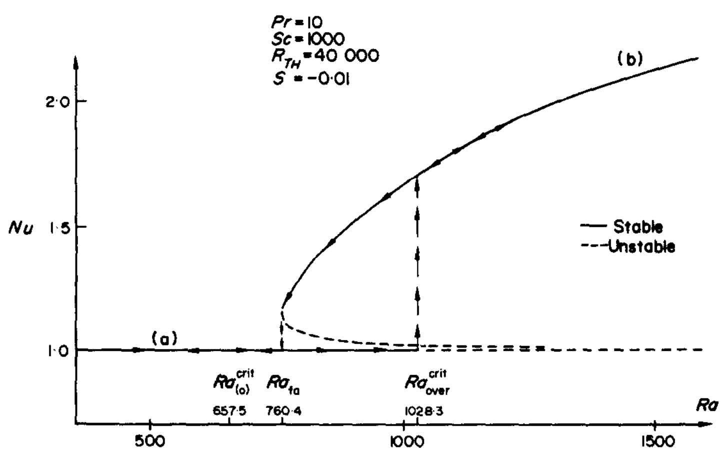

FIG. 2. Hysteresis loop in the Nusselt–Rayleigh number plane.

In the notation $Ra_{\text{f.a.}} = 760{\cdot}4$, the subscript "f.a." means a Rayleigh at which finite amplitude instability *may* exist.

Curve (b) cuts curve (a) at $Ra = Ra_{\text{ex}}^{\text{crit}}$, which is a bifurcation point. There is a striking analogy with recent results of Nicolis and Auchmuty [20] in their study of steady solutions of nonlinear equations describing a chemical network.

The next step is to test the stability of each solution for A against infinitesimal disturbances around a given steady state, solution of equations (15)–(19). Equations (10)–(14) are linearized in the perturbations $\delta A, \delta B \ldots$ of $A, B \ldots$.

The time dependence of each perturbation is then given by $e^{\omega t}$ and we have then to find the eigenvalues ω of a 5×5 real matrix. If real $\{\omega\} > 0$ for at least one eigenvalue, the steady state considered is unstable. The stability of $A = 0$ (state of rest; curve (a) of Fig. 2) was already studied in an earlier paper [2]. This solution is stable if $Ra < Ra_{\text{over}}^{\text{crit}}$ (in the present numerical example if $Ra < 1028{\cdot}35$). This is indicated in Fig. 2 by a full line. When $Ra > 1028{\cdot}35$, a perturbation with $r^2 = \frac{1}{2}$, grows exponentially with time (ω is complex!), and this is indicated in Fig. 2 by a dotted line (unstable solution). The stability of each solution on curve (b) was also tested. We have found that the part of curve (b) with a negative slope was always unstable (lower branch) and that the upper branch was stable. Thus if $Ra_{\text{f.a.}} < Ra < Ra_{\text{over}}^{\text{crit}}$, two *stable* steady states are indeed observed. The solution chosen by the system will depend upon the initial conditions. The system of equations (10)–(14) is integrated numerically for different initial conditions. Table 1 summarizes the results of the numerical integration. Our results clearly show that multiple steady state are possible below the critical point. Thus the existence of an hysteresis loop has been numerically shown (paths indicated by arrows).

This result must be compared with Schmidt–Milverton plots for a two component system where, below the critical temperature difference (or Rayleigh number) multiple states (a state of rest and convective states) were indeed observed (see Fig. 1).

The variation of $Ra_{\text{f.a.}}$ with $\mathscr{S}$ can also be given. Indeed $Ra_{\text{f.a.}}$ is the value of Ra which makes zero the quantity under the square root sign in equation (24). Thus

$$\frac{9}{4}\frac{1}{Pr^4}\left[\frac{Ra}{Ra^{(0)}} - \left(1 + \frac{Pr^2}{Sc^2}\right)\right]^2 + \left(\frac{Ra - Ra_{\text{ex}}^{\text{crit}}}{Ra^{(0)}}\right) \times \frac{9}{Sc^2 Pr^2} = 0 \quad (26)$$

or using equation (21)

$$\frac{Ra_{\text{f.a.}}}{Ra^{(0)}} = 1 - \left(\frac{Pr}{Sc}\right)^2 \pm 2\left(\frac{Pr}{Sc}\right) \times \sqrt{\left\{-\frac{R_{Th}\cdot\mathscr{S}}{Ra^{(0)}}\left[1 + \left(\frac{Pr}{Sc}\right)^{-1}\right]\right\}}. \quad (27)$$

As already stated, and now shown by equation (27) finite amplitude convection can only exist for $\mathscr{S} < 0$.

Table 2 gives the three Rayleigh numbers of interest for different $\mathscr{S}$ and $Pr = 10$; $Sc = 1000$; $R_{Th} = 40\,000$ and shows that finite amplitude instability always exist for $\mathscr{S} < 0$.

Table 2

Soret number	$Ra_{\text{ex}}^{\text{crit}}$	$Ra_{\text{over}}^{\text{crit}}$	$Ra_{\text{f.a.}}$
-10^{-7}	657·9	664·7	657·8
-10^{-6}	·661·5	664·8	658·5
-10^{-5}	697·9	665·1	660·7
-10^{-4}	1061·5	668·4	667·8
-10^{-3}	4697·5	701·1	690·0
-5×10^{-3}	20857·5	846·6	730·3
-10^{-2}	41057·5	1028·4	760·5
-2×10^{-2}	81457·5	1392·0	803·2
-4×10^{-2}	162257·5	2119·3	863·6
-6×10^{-2}	243057·5	2846·6	909·9
-8×10^{-2}	323857·5	3573·8	949·0
-10^{-1}	404657·5	4301·1	983·4

CONCLUSION

An approximate nonlinear analysis shows that multiple steady state exist below the critical Rayleigh number. Two of them are stable: the state of rest is stable if the disturbances are sufficiently small but is unstable if they are sufficiently large. The system evolves

Table 1

Ra	Initial conditions	Final state
$Ra < 1028{\cdot}35$	State of rest ($A=B=C=D=E=0$) + small perturbation ($B = C = -10^{-6}$) $Nu(t=0) = 1{\cdot}000006$	State of rest $Nu(t\to\infty) = 1{\cdot}000000$
$Ra = 1257{\cdot}5$ ($Ra > Ra_{\text{over}}^{\text{crit}}$)	State of rest + small perturbation $Nu(t=0) = 1{\cdot}000006$	Steady convective state. Integration performed for $0 < t < 3000$ (time step: 10^{-2}) $t = 3000 \simeq \infty$ $Nu(t=3000) = 1{\cdot}950$
$Ra = 767{\cdot}5$ $Ra_{\text{f.a.}} < Ra < Ra_{\text{over}}^{\text{crit}}$	Final steady state obtained for $Ra = 1257{\cdot}5$ $Nu(t=0) = 1{\cdot}950$	New steady convective state $0 < t < 900 \simeq \infty$ $Nu(t=900) = 1{\cdot}203$

towards a new steady convective state, even below the critical point, and in turn this new steady state is stable against infinitesimal disturbances. This situation prevails for negative Soret coefficients. It seems thus that we have explained one of the anomalous behaviour of a multicomponent system heated from below with a negative thermal diffusion factor. To our knowledge, this applies to the following systems: water–methanol; water–ethanol; water–isopropanol (90 wt % water); sea water; some electrolyte solutions.

Presently a complete numerical study using as much as 120 Fourier coefficients is in progress.

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UN CYCLE D'HYSTERESIS DANS LE PROBLEME DE BENARD A DEUX COMPOSANTS

Résumé—Une approximation de la théorie non-linéaire du problème de Bénard à deux composants qui tient compte de la diffusion thermique montre que des états stationnaires multiples stables sont possibles. La stabilité de chaque état stationnaire est examinée. Les équations non-linéaires sont ensuite intégrées numériquement. On observe un cycle d'hystérésis dans le plan nombre de Rayleigh–nombre de Nusselt.

HYSTERESE BEIM ZWEIKOMPONENTEN BENARD-PROBLEM

Zusammenfassung—Eine Näherungslösung der nichtlinearen Theorie des Zweikomponenten Bénard-Problems unter Berücksichtigung der Thermodiffusion zeigt, daß mehrfach stabile stetige Zustände möglich sind. Die Stabilität eines jeden stetigen Zustandes wurde untersucht. Die nichtlinearen Gleichungen wurden numerisch integriert. Beim Auftragen der Rayleigh-Zahl über der Nusselt-Zahl wurde eine Hysterese festgestellt.

ПЕТЛЯ ГИСТЕРЕЗИСА В ДВУХКОМПОНЕНТНОЙ ЗАДАЧЕ БЕНАРА

Аннотация — Аппроксимация нелинейной теории двухкомпонентной задачи Бенара при учете термодиффузии показывает, что возможно множество устойчивых стационарных состояний. Исследуется устойчивость каждого стационарного состояния. Затем проводится численное интегрирование нелинейных уравнений. Петля Гистерезиса наблюдается в плоскости число Релея — число Нуссельта.